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Mathematics

Upper Secondary Level

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Young Digital Poland
EDUCATION TECHNOLOGIES

Mathematics

Upper Secondary level

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Graphical interpretation of algebraic solutions
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Use of intersection points

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$$a^p \cdot a^q = a^{p+q}$$

$$a^p \cdot b^p = (ab)^p$$

$$\frac{a^p}{a^q} = a^{p-q} \text{ if } a \neq 0, \frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p \text{ if } b \neq 0, (a^p)^q = a^{pq} \text{ for } p,$$

q rational

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Domains
Ranges
Graphs

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